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Radiative generation of the Higgs potential

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Abstract

We consider the minimal extension of the Standard Model with $U(1)_{B-L}$ gauge symmetry for generating the Higgs potential radiatively. Assuming that the full scalar potential vanishes at the vacuum instability scale, we achieve the goal in terms of two free parameters, the $B-L$ gauge coupling and the right-handed neutrino Yukawa coupling. The $B-L$ gauge symmetry is broken spontaneously by the Coleman-Weinberg mechanism while the scale symmetry breakdown induces electroweak symmetry breaking through the radiative generation of appropriate scalar quartic couplings. We show that there is a reasonable parameter space that is consistent with a correct electroweak symmetry breaking and the observed Higgs mass.

1 Introduction

The establishment of the Standard Model (SM), that requires a hypercharged doublet boson H under the SM gauge symmetry $SU(2)_L \times U(1)_Y$ as the origin of the electroweak symmetry breaking (EWSB) and all the quark and charged lepton masses, has culminated by the discovery of a new boson at a mass around 126 GeV [1]. We now have fairly good information on the SM scalar potential describing the EWSB mechanism:

$$V_H = m_H^2 |H|^2 + \lambda_H |H|^4 \tag{1}$$

where H can be written as $H = (0, v_{EW} + h)/\sqrt{2}$ in the unitary gauge. The vacuum expectation value v_{EW} and the Higgs particle mass m_h are determined by the two input parameters m_H and λ_H through the relation:

$$m_H^2 = -\lambda_H v_{EW}^2 = -\frac{1}{2} M_h^2\,, \tag{2}$$

which tells us $m_H^2 \approx -(89\,\mathrm{GeV})^2$ and $\lambda_H \approx 0.13$ translated from the observed values of $v_{EW} \simeq 246$ GeV and $M_h \approx 126$ GeV.

The value of $\lambda_H \approx 0.13$ at the electroweak scale is quite intriguing as it is driven to zero at a large scale close to the Planck scale [2] and develops the vacuum instability [3]. This may tell us about a possibility of generating the Higgs quartic coupling radiatively starting from the vanishing initial condition at a high scale [4]. The electroweak scale v_{EW} (or m_H^2) might also have a radiative origin as was shown decades ago by Coleman and Weinberg [5] that a mass scale can be generated via dimensional transmutation from running couplings in a massless scalar gauge theory. Combining these two features, one may envisage a possibility that the whole Higgs potential (1) is dynamically generated.

While the Coleman-Weinberg (CW) mechanism does not work for the SM, it is conceivable to extend the SM with an additional scalar $U(1)$ gauge theory in which the additional $U(1)$ symmetry is spontaneously broken by radiative corrections and the generated mass scale is transferred to the SM [6–10]. Thus, in this paper, we consider the possibility of implementing the CW mechanism to the $U(1)_{B-L}$ gauge symmetry [8], which is a natural extension of the SM with three right-handed neutrinos to explain the observed neutrino masses and mixing.

The $B-L$ extended SM contains an additional complex scalar field Φ whose vacuum expectation value (VEV) breaks $U(1)_{B-L}$ and induces Majorana masses of right-handed neutrinos $M_N = y_N \langle\Phi\rangle$ through the Yukawa coupling y_N. Once a non-vanishing $\langle\Phi\rangle$ is generated, it can generate the Higgs mass by $m_H^2 = \lambda_{H\Phi}\langle\Phi\rangle^2$ in the presence of the mixing potential term $\lambda_{H\Phi}|\Phi|^2|H|^2$. Amusingly, the mixing coupling $\lambda_{H\Phi}$ is generated at two loops through the radiative generation of the kinetic mixing between the $U(1)_Y$ and $U(1)_{B-L}$ gauge fields [9]. As will be seen, it is remarkable that the coupling λ_Φ of the quartic potential term $\lambda_\Phi|\Phi|^4$ is radiatively generated by the right-handed neutrino Yukawa coupling y_N in a similar way as in the Higgs quartic coupling in the SM. But, there is a difference that the beta function of λ_Φ changes sign during the renormalization group

(RG) evolution, unlike the monotonic behavior of the running Higgs quartic coupling λ_H due to the top Yukawa coupling. That is, λ_Φ is generated radiatively below the instability scale dominantly by a sizable right-handed neutrino Yukawa coupling but it becomes small enough for the Coleman-Weinberg mechanism to work at even lower scales.

Putting together all the features discussed above, we wish to entertain a paradigm of radiative generation of all the scalar potential terms which are supposed to vanish at a certain UV scale. To be specific, we consider the possibility of achieving a spontaneous breaking of the electroweak and $B-L$ gauge symmetries by the addition of only two free parameters, the right-handed neutrino Yukawa coupling y_N and the $B-L$ gauge coupling g_{B-L}, in the $B-L$ extended SM.

The paper is organized as follows. We begin with the description of the $B-L$ extension of the SM and review the Coleman-Weinberg potential in the model. Then we perform the RG analysis of the model and search the parameter space that give rises to a correct electroweak symmetry breaking and Higgs mass. Finally, conclusions are drawn. There is one appendix containing the RG equations of the model.

2 $B-L$ extension of the SM and Coleman-Weinberg potential

The $B-L$ extension of the Standard Model that we are considering is described by the following Lagrangian [11],

$$\mathcal{L} = \mathcal{L}_S + \mathcal{L}_{\rm YM} + \mathcal{L}_F + \mathcal{L}_Y, \tag{3}$$

with

$$\begin{aligned} \mathcal{L}_S &= |D_\mu H|^2 + |D_\mu \Phi|^2 - m_H^2 |H|^2 - m_\Phi^2 |\Phi|^2 - \lambda_H |H|^4 - \lambda_\Phi |\Phi|^4 - \lambda_{H\Phi} |H|^2 |\Phi|^2, & (4) \\ \mathcal{L}_{\rm YM} &= -\frac{1}{4}(F^{\mu\nu}F_{\mu\nu})_{\rm SM} - \frac{1}{4}F'^{\mu\nu}F'_{\mu\nu}, & (5) \\ \mathcal{L}_F &= i\bar{q}_L \not{D} q_L + i\bar{u}_R \not{D} u_R + i\bar{d}_R \not{D} d_R + i\bar{l}_L \not{D} l_L + i\bar{e}_R \not{D} e_R + i\bar{\nu}_R \not{D} \nu_R, & (6) \\ \mathcal{L}_Y &= -y_d \bar{q}_L d_R H - y_u \bar{q}_L u_R \tilde{H} - y_e \bar{l}_L e_R H - y_\nu \bar{l}_L \nu_R \tilde{H} - y_N \overline{(\nu_R)^c} \nu_R \Phi & (7) \end{aligned}$$

where $\tilde{H} = i\sigma^2 H^*$ and the covariant derivative is

$$D_\mu = \partial_\mu + i g_S T^\alpha G_\mu^\alpha + i g T^a W_\mu^a + i g_Y Y B_\mu + i(\tilde{g} Y + g_{B-L} Y_{B-L}) B'_\mu. \tag{8}$$

Note that the gauge coupling $\tilde{g}$ describes the kinetic mixing between $U(1)_Y$ and $U(1)_{B-L}$. When the complex scalar Φ, carrying a $B-L$ number 2, gets a VEV, the $B-L$ gauge symmetry is broken spontaneously and the right-handed neutrinos obtain masses.

Now we consider the one-loop Coleman-Weinberg potential [5] for the $B-L$ sector. In the limit of $|\lambda_{H\Phi}|, |\tilde{g}| \ll 1$ at tree level, we can ignore the SM contributions to the CW potential and focus on the $B-L$ sector. Taking $\Phi = \phi/\sqrt{2}$ in the unitary gauge and

imposing the renormalization conditions [5],

$$\left.\frac{\partial^2 V}{\partial \phi^2}\right|_{\phi=0} = 0, \tag{9}$$

$$\left.\frac{\partial^4 V}{\partial \phi^4}\right|_{\phi=M} = 6\lambda_\Phi, \tag{10}$$

the one-loop corrected $B-L$ potential becomes

$$V_{B-L}(\phi) = \frac{1}{4}\lambda_\Phi \phi^4 + \frac{\phi^4}{64\pi^4}\Big(10\lambda_\Phi^2 + 48 g_{B-L}^2 - \frac{1}{2}\sum_{i=1}^{3} y_{N_i}^4\Big)\Big(\ln\frac{\phi^2}{M^2} - \frac{25}{6}\Big) \tag{11}$$

where we assumed that the Yukawa couplings for the right-handed neutrinos are diagonal as $y_{N,ij} = y_{N_i}\delta_{ij}$. For our analysis in the following, we will take only one coupling y_N.

Choosing the renormalization scale at $M = \langle\phi\rangle \equiv v_\phi$ to avoid the large-log uncertainty in the one-loop approximation [5], one can evaluate the minimization condition of the potential (11) and obtain

$$\lambda_\Phi(v_\phi) = \frac{11}{48\pi^2}\Big(10\lambda_\Phi^2 + 48 g_{B-L}^4 - \frac{1}{2}y_N^4\Big)(v_\phi). \tag{12}$$

This relation fixes the $B-L$ breaking scale v_ϕ in terms of input values of λ_Φ, g_{B-L} and y_N which evolve from a high scale M_* to v_ϕ by RG. As a consequence, the CW potential (11) leads to a naturally small VEV v_ϕ via dimensional transmutation as

$$v_\phi \simeq M_*\, e^{\frac{11}{6}} \exp\Big(-\frac{\pi^2}{6}\frac{\lambda_\Phi(M_*)}{g_{B-L}^4(M_*) - \frac{1}{96}y_N^4(M_*)}\Big) \tag{13}$$

where the small λ_Φ^2 contribution is neglected and $g_{B-L}^4 - \frac{1}{96}y_N^4 > 0$. When the beta function of λ_Φ changes the sign during the RG evolution, we should take M_* to be below the scale where $g_{B-L}^4 - \frac{1}{96}y_N^4 = 0$. As will be seen in the later section, even if we start with a vanishing λ_Φ at the cutoff scale, a positive λ_Φ is generated at a smaller scale by the RG evolution with a positive beta function of λ_Φ, setting the initial couplings for dimensional transmutation given in eq. (13).

We also obtain the physical masses of the $B-L$ scalar ϕ and the gauge boson Z_{B-L} in the vacuum as

$$M_\phi^2 = \frac{6}{11}\lambda_\Phi(v_\phi)v_\phi^2 \quad \text{and} \quad M_{B-L}^2 = 4 g_{B-L}^2(v_\phi)v_\phi^2\,, \tag{14}$$

which determines the ratio $M_\phi^2/M_{B-L}^2 \approx 3g_{B-L}^2/2\pi^2$ putting a rough relation of $\lambda_\Phi \approx 11 g_{B-L}^4/\pi^2$.

Note that we need $\lambda_\Phi \sim g_{B-L}^4$ for the CW mechanism to work in the $B-L$ sector, and $|\lambda_{H\Phi}| \sim M_Z^2/v_\phi^2$ for the EWSB. Thus, the required quartic couplings amount to $\lambda_\Phi \sim 10^{-2}$ and $|\lambda_{H\Phi}| \lesssim 10^{-3}$, taking $g_{B-L} \sim 0.3$ and $v_\phi \gtrsim 3$ TeV. As will be shown in the next section, such small values can naturally be generated radiatively starting from the vanishing initial condition at a high scale.

3 Radiative $B-L$ and electroweak symmetry breaking

The discovered Higgs boson has a mass at 126 GeV, so the Higgs quartic coupling vanishes at M_I, the so called vacuum instability scale[1], which is below the Planck scale, for the top pole mass $M_t > 171$ GeV [2]. If there is a sizable new physics contribution to the running of the Higgs quartic coupling, it is possible to increase the vacuum instability scale. But, in order not to reintroduce the hierarchy problem, a new particle mass curing the vacuum instability should not be far away from the weak scale [12]. In our case, we assume that there is no heavy particle other than the $B-L$ sector. Then, the vacuum instability scale remains the same as in the SM.

Generalizing the vanishing feature of the Higgs quartic coupling at M_I, we assume the initial condition of vanishing all the scalar potential terms as well as the kinetic mixing coupling $\tilde{g}$ in a certain UV completed theory at M_I, and examine whether it leads to viable electroweak and $B-L$ symmetry breaking by solving the RG equations presented in the Appendix. That is, we impose the initial condition at M_I:

$$\lambda_H = 0,\ \lambda_\Phi = 0,\ \lambda_{H\Phi} = 0,\ m_\Phi^2 = 0,\ m_H^2 = 0,\ \text{and}\ \tilde{g} = 0, \tag{15}$$

with arbitrary values of y_N and g_{B-L}, and then calculate the RG generated values at lower scale to determine the $B-L$ breaking scale v_ϕ at which the relation (12) is satisfied. During the RG evolution, a small negative $\lambda_{H\Phi}$ is generated at two loops involving $\tilde{g}^2 g_{B-L}^2$ and thus the Higgs potential develops at v_ϕ as

$$V_{\rm SM}(h) = \frac{1}{2} m_H^2(v_\phi) h^2 + \frac{1}{4}\lambda_H(v_\phi) h^4 \tag{16}$$

with

$$m_H^2(v_\phi) = \frac{1}{2}\lambda_{H\Phi}(v_\phi) v_\phi^2\,. \tag{17}$$

Below v_ϕ, the Higgs quartic coupling λ_H runs approximately as in the SM while the running of the Higgs mass parameter is governed by

$$\frac{dm_H^2}{d\ln\mu} = \frac{1}{16\pi^2}\Big[m_H^2\Big(12\lambda_H + 6y_t^2 - \frac{9}{2}g^2 - \frac{3}{2}g_Y^2 - \frac{3}{2}\tilde{g}^2\Big) + 2\lambda_{H\Phi} M_\phi^2\Big] \tag{18}$$

where M_ϕ^2 is given in eq. (14). Note that the right-hand side of (18) contains an additive term proportional to a new scalar mass-squared M_ϕ^2 given by (14). But it gives a contribution of order $\lambda_\Phi m_H^2$ which is negligible for $\lambda_\Phi \ll 1$. Then, the electroweak VEV is determined by

$$v_{EW} = \sqrt{-\frac{m_H^2(v_{EW})}{\lambda_H(v_{EW})}}. \tag{19}$$

[1]The vacuum instability scale is essentially regarded as the UV cutoff but we don't specify a UV completion for curing the vacuum instabilty. It could be a field-theoretical UV completion with heavy particle or a quantum gravity. The former case could reintroduce the hierarchy problem via the Higgs coupling.

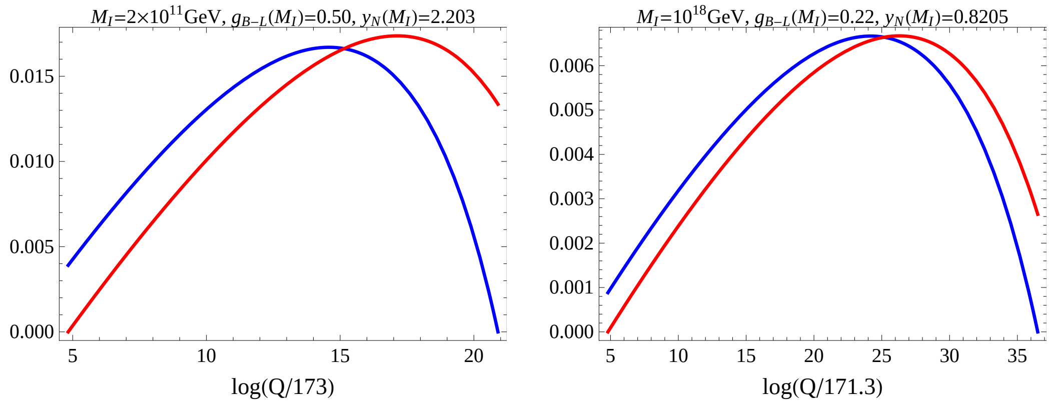

Figure 1: Examples of running $B-L$ scalar quartic coupling λ_Φ (Blue), and the $B-L$ minimization condition (12) (Red) for the instability scale $M_I = 2\times 10^{11}$ GeV on the left and $M_I = 10^{18}$ GeV on the right. Successful electroweak symmetry occurs in both examples.

On the other hand, the Higgs mass is given by

$$M_h^2 = 2\lambda_H(v_{\rm EW})v_{EW}^2 + \Delta M_h^2 \tag{20}$$

where ΔM_h^2 is the Higgs self-energy correction to the Higgs pole mass [2]. Thus, one can select out appropriate initial values of y_N and g_{B-L} reproducing two observables, $v_{EW} \simeq 246$ GeV and $M_h \approx 126$ GeV. We consider two UV scales, $M_I = 2\times 10^{11}$ and 10^{18} GeV, corresponding to the vacuum instability scales for the top mass $M_t = 173$ and 171.3 GeV, respectively.

Figure 1 shows examples of radiative generation of the $B-L$ quartic coupling λ_Φ and the $B-L$ symmetry breaking scale at $v_\phi = M_t e^5$ for appropriate choices of y_N and g_{B-L} at $M_I = 2\times 10^{11}$ and 10^{18} GeV, respectively. In both examples, the beta function of λ_Φ starts with a negative sign at the instability scale so a positive λ_Φ is generated at a smaller scale, setting the initial parameters for a dimensional transmutation via eq. (13). But, the beta function of λ_Φ at even smaller scales becomes positive, resulting in a small λ_Φ appropriate for satisfying the $B-L$ minimization condition.

The results of our analysis are summarized in Figure 2. It shows the lines in the plane of the $B-L$ gauge coupling g_{B-L} and the right-handed neutrino Yukawa coupling y_N determined at the $B-L$ symmetry breaking scale v_ϕ, which satisfy the electroweak symmetry breaking conditions. Also shown are the induced values of λ_Φ and v_ϕ as functions of g_{B-L}. The upper (lower) panels are for $M_I = 2\times 10^{11}$ (10^{18}) GeV. From these plots, one can read off the predicted mass scales: $M_{B-L} = 2g_{B-L}v_\phi$ for the $B-L$ gauge boson, $M_\phi = \sqrt{6\lambda_\Phi/11}v_\phi$ for the $B-L$ scalar, and $M_N = \sqrt{2}y_N v_\phi$ for the right-handed neutrino. We find that in both stability scales, M_{B-L} and M_N can be multi-TeV while M_ϕ can be at sub-TeV. The $B-L$ breaking scale is lower for larger g_{B-L} and for higher M_I, so we

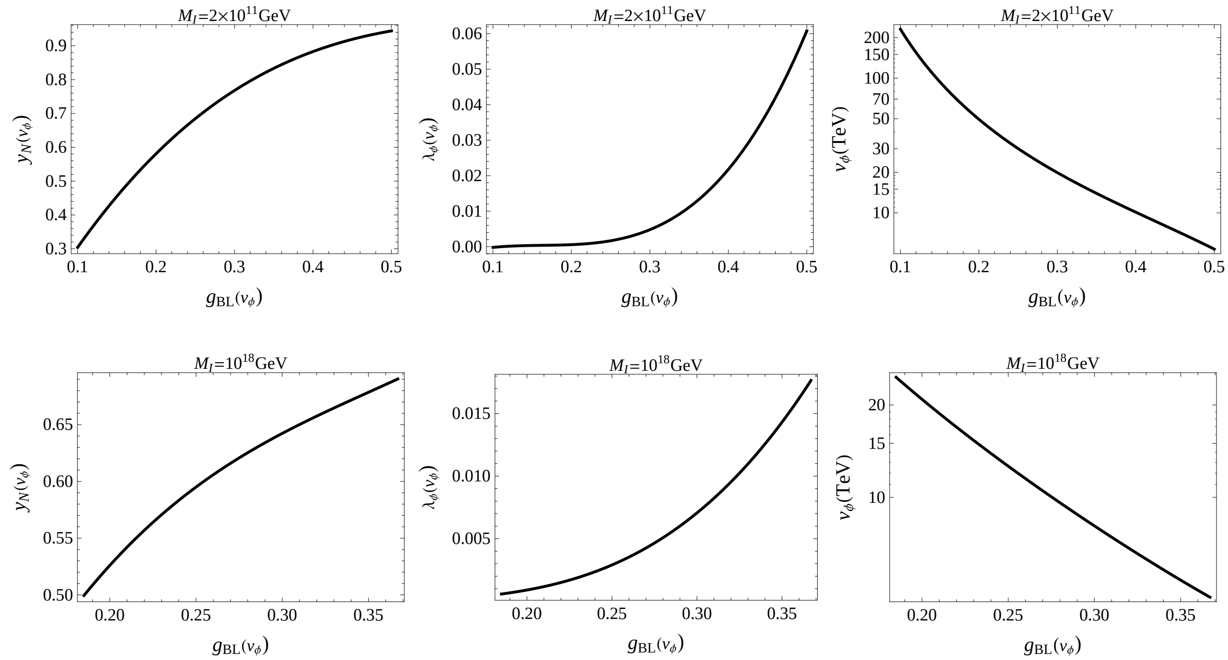

Figure 2: The values of the $B-L$ gauge coupling g_{B-L} vs. the right-handed neutrino Yukawa coupling y_N, the $B-L$ scalar quartic coupling λ_Φ, and the $B-L$ breaking scale v_ϕ realizing successful electroweak symmetry breaking. We have chosen the Higgs mass at 126 GeV and the instability scale, $M_I = 2 \times 10^{11}$ GeV and 10^{18} GeV, in the upper and lower panels, respectively.

need a smaller g_{B-L} at the $B-L$ breaking scale in the case of $M_I = 10^{18}$ GeV than in the case of $M_I = 2 \times 10^{11}$ GeV. We note that v_ϕ around a few TeV can be obtained for $g_{B-L}(v_\phi) \gtrsim 0.35$ ($g_{B-L}(M_I) \gtrsim 0.5$) for which the $B-L$ gauge boson signatures may be found in the future LHC run. One can see from Figure 2 that the radiative breaking of the $B-L$ and electroweak symmetries occurs appropriately in a reasonable range of the two free parameters y_N and g_{B-L}.

Let us finally comment on the possibility of allowing the right-handed neutrino mass higher than 10^9 GeV for which successful thermal leptogenesis can easily occur [13]. In order to achieve the EWSB with a high $B-L$ breaking scale $v_\phi > 10^9$ GeV, one needs to generate an extremely small mixing coupling $|\lambda_{H\Phi}| < 10^{-14}$. This requires an unnaturally small value of $g_{B-L} \lesssim 10^{-3}$ as the RG induces roughly $|\lambda_{H\Phi}| \sim g_{B-L}^4 g_Y^4 \ln^3(M_I/v_\phi)/\pi^4$.

4 Conclusion

The origin of the EWSB would be a fundamental question in the SM, which may be related to new physics explaining neutrino masses and/or dark matter. An appealing way of generating a mass scale dynamically is the Coleman-Weinberg mechanism which

is known to work in a massless scalar $U(1)$ gauge theory. It is then tempting to consider a $U(1)$ symmetry in connection with neutrino mass generation and/or the existence of dark matter. As one of the examples, we considered the radiative breaking of the $B-L$ symmetry which can lead to right-handed neutrino masses and the Higgs mass at the same time. We note that the $B-L$ scalar quartic coupling, although vanishing at a high scale, can be generated dynamically due to the right-handed neutrino Yukawa coupling y_N, just as the SM scalar quartic coupling is generated due to the top quark Yukawa coupling below the instability scale.

Performing the RG analysis in the $B-L$ extended SM, we examined the possibility of generating radiatively the whole scalar potential at low scales for the initial condition of a vanishing potential at the instability scale. A small quartic coupling λ_Φ required for the Coleman-Weinberg generation of the $B-L$ scalar VEV is obtained even for a vanishing initial λ_Φ due to the sign change of the corresponding beta function. We also found that a naturally small mixing coupling of the $B-L$ and Higgs scalars is generated through the radiative kinetic mixing between $U(1)_{B-L}$ and $U(1)_Y$ gauge bosons driven by $B-L$ and SM charged quarks and leptons. We showed that there are reasonable values of y_N and g_{B-L} for the successful $B-L$ and electroweak symmetry breaking, being consistent with the measured Higgs boson mass.

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Appendix A: Renormalization group equations

The running of couplings p_i are governed by the RG equations, $\frac{dp_i}{dt} = \beta_{p_i}$ with β_{p_i} being the corresponding beta functions and $t \equiv \ln(Q/M_t)$. We present the beta functions in the $B-L$ extension of the SM [11].

First, the one-loop beta functions of the gauge couplings are

$$(4\pi)^2\beta_{g_Y} = \frac{41}{6}g_Y^3, \quad (4\pi)^2\beta_g = -\frac{19}{6}g^3, \quad (4\pi)^2\beta_{g_S} = -7g_S^3, \tag{A.1}$$

$$(4\pi)^2\beta_{g_{B-L}} = 12g_{B-L}^3 + \frac{32}{3}g_{B-L}^2\,\tilde{g} + \frac{41}{6}g_{B-L}\,\tilde{g}^2, \tag{A.2}$$

$$(4\pi)^2\beta_{\tilde{g}} = 41\tilde{g}(\tilde{g}^2 + 2g_Y^2) + \frac{32}{3}g_{B-L}(\tilde{g}^2 + g_Y^2) + 12g_{B-L}^2\,\tilde{g}. \tag{A.3}$$

Next, the one-loop beta functions for top Yukawa coupling and the Yukawa couplings of the right-handed neutrinos are

$$(4\pi)^2\beta_{y_t} = y_t\Big(\frac{9}{2}y_t^2 - 8g_S^2 - \frac{9}{4}g^2 - \frac{17}{12}g_Y^2 - \frac{17}{12}\tilde{g}^2 - \frac{2}{3}g_{B-L}^2 - \frac{5}{3}\tilde{g}\,g_{B-L}\Big), \tag{A.4}$$

$$(4\pi)^2\beta_{y_{N_i}} = y_{N_i}\Big(4y_{N,i}^2 + 2\mathrm{Tr}(y_N^2) - 6g_{B-L}^2\Big). \tag{A.5}$$

Finally, the one-loop beta functions of the parameters in the potential are

$$(4\pi)^2\beta_{m_H^2} = m_H^2\Big(12\lambda_H + 6y_t^2 - \frac{9}{2}g^2 - \frac{3}{2}g_Y^2 - \frac{3}{2}\tilde{g}^2\Big) + 2\lambda_{H\Phi}m_\Phi^2, \tag{A.6}$$

$$(4\pi)^2\beta_{m_\Phi^2} = m_\Phi^2\Big(8\lambda_\Phi + 4\mathrm{Tr}(y_N^2) - 24g_{B-L}^2\Big) + 4\lambda_{H\Phi}m_H^2, \tag{A.7}$$

and

$$\begin{aligned}(4\pi)^2\beta_{\lambda_H} &= 24\lambda_H^2 - 6y_t^2 + \frac{9}{8}g^4 + \frac{3}{8}g_Y^4 + \frac{3}{4}g^2g_Y^2 + \frac{3}{4}g^2\tilde{g}^2 + \frac{3}{8}\tilde{g}^4 \\ &\quad +\lambda_H(12y_t^2 - 9g^2 - 3g_Y^2 - 3\tilde{g}^2) + \lambda_{H\Phi}^2,\end{aligned} \tag{A.8}$$

$$(4\pi)^2\beta_{\lambda_\Phi} = 20\lambda_\Phi^2 - \mathrm{Tr}(y_N^4) + 96g_{B-L}^4 + 8\lambda_\Phi\mathrm{Tr}(y_N^2) - 48\lambda_\Phi g_{B-L}^2 + 2\lambda_{H\Phi}^2, \tag{A.9}$$

$$\begin{aligned}(4\pi)^2\beta_{\lambda_{H\Phi}} &= \lambda_{H\Phi}\Big(12\lambda_H + 8\lambda_\Phi + 4\lambda_{H\Phi} + 6y_t^2 - \frac{9}{2}g^2 - \frac{3}{2}g_Y^2 - \frac{3}{2}\tilde{g}^2 \\ &\quad +4\mathrm{Tr}(y_N^2) - 24g_{B-L}^2\Big) + 12\tilde{g}^2g_{B-L}^2.\end{aligned} \tag{A.10}$$

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